7.7 Solve Right Triangles

Before You used tangent, sine, and cosine ratios.

Now You will use inverse tangent, sine, and cosine ratios.

Why? So you can build a saddlerack, as in Ex. 39.

Key Vocabulary
- solve a right triangle
- inverse tangent
- inverse sine
- inverse cosine

To **solve a right triangle** means to find the measures of all of its sides and angles. You can solve a right triangle if you know either of the following:

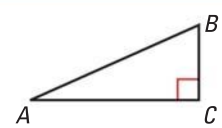
- Two side lengths
- One side length and the measure of one acute angle

In Lessons 7.5 and 7.6, you learned how to use the side lengths of a right triangle to find trigonometric ratios for the acute angles of the triangle. Once you know the tangent, the sine, or the cosine of an acute angle, you can use a calculator to find the measure of the angle.

KEY CONCEPT *For Your Notebook*

Inverse Trigonometric Ratios

Let $\angle A$ be an acute angle.



Inverse Tangent If $\tan A = x$, then $\tan^{-1} x = m\angle A$. $\quad \tan^{-1}\frac{BC}{AC} = m\angle A$

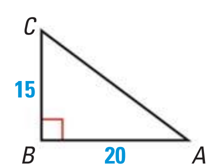
Inverse Sine If $\sin A = y$, then $\sin^{-1} y = m\angle A$. $\quad \sin^{-1}\frac{BC}{AB} = m\angle A$

Inverse Cosine If $\cos A = z$, then $\cos^{-1} z = m\angle A$. $\quad \cos^{-1}\frac{AC}{AB} = m\angle A$

READ VOCABULARY
The expression "$\tan^{-1}x$" is read as "the inverse tangent of x."

EXAMPLE 1 Use an inverse tangent to find an angle measure

Use a calculator to approximate the measure of $\angle A$ to the nearest tenth of a degree.



Solution

Because $\tan A = \frac{15}{20} = \frac{3}{4} = 0.75$, $\tan^{-1} 0.75 = m\angle A$. Use a calculator.

$\tan^{-1} 0.75 \approx 36.86989765 \cdots$

▶ So, the measure of $\angle A$ is approximately 36.9°.

EXAMPLE 2 Use an inverse sine and an inverse cosine

ANOTHER WAY
You can use the Table of Trigonometric Ratios on p. 925 to approximate $\sin^{-1} 0.87$ to the nearest degree. Find the number closest to 0.87 in the sine column and read the angle measure at the left.

Let $\angle A$ and $\angle B$ be acute angles in a right triangle. Use a calculator to approximate the measures of $\angle A$ and $\angle B$ to the nearest tenth of a degree.

a. $\sin A = 0.87$ **b.** $\cos B = 0.15$

Solution

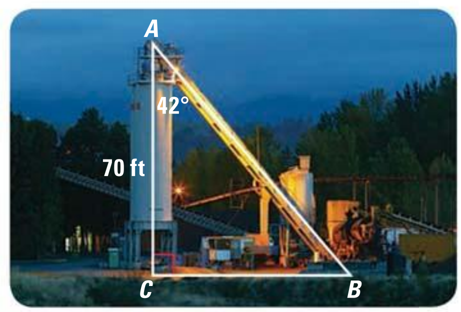
a. $m\angle A = \sin^{-1} 0.87 \approx 60.5°$ **b.** $m\angle B = \cos^{-1} 0.15 \approx 81.4°$

✓ GUIDED PRACTICE for Examples 1 and 2

1. Look back at Example 1. Use a calculator and an inverse tangent to approximate $m\angle C$ to the nearest tenth of a degree.
2. Find $m\angle D$ to the nearest tenth of a degree if $\sin D = 0.54$.

EXAMPLE 3 Solve a right triangle

Solve the right triangle. Round decimal answers to the nearest tenth.



Solution

STEP 1 **Find** $m\angle B$ by using the Triangle Sum Theorem.

$180° = 90° + 42° + m\angle B$

$48° = m\angle B$

STEP 2 **Approximate** BC by using a tangent ratio.

$\tan 42° = \frac{BC}{70}$ Write ratio for tangent of 42°.

$70 \cdot \tan 42° = BC$ Multiply each side by 70.

$70 \cdot 0.9004 \approx BC$ Approximate tan 42°.

$63 \approx BC$ Simplify and round answer.

ANOTHER WAY
You could also find *AB* by using the Pythagorean Theorem, or a sine ratio.

STEP 3 **Approximate** AB using a cosine ratio.

$\cos 42° = \frac{70}{AB}$ Write ratio for cosine of 42°.

$AB \cdot \cos 42° = 70$ Multiply each side by *AB*.

$AB = \frac{70}{\cos 42°}$ Divide each side by cos 42°.

$AB \approx \frac{70}{0.7431}$ Use a calculator to find cos 42°.

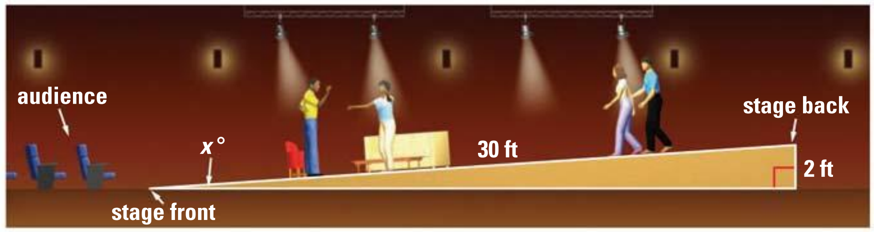
$AB \approx 94.2$ Simplify .

▶ The angle measures are 42°, 48°, and 90°. The side lengths are 70 feet, about 63 feet, and about 94 feet.

EXAMPLE 4 Solve a real-world problem

READ VOCABULARY
A *raked stage* slants upward from front to back to give the audience a better view.

THEATER DESIGN Suppose your school is building a *raked stage*. The stage will be 30 feet long from front to back, with a total rise of 2 feet. A rake (angle of elevation) of 5° or less is generally preferred for the safety and comfort of the actors. Is the raked stage you are building within the range suggested?



Solution

Use the sine and inverse sine ratios to find the degree measure x of the rake.

$$\sin x° = \frac{\text{opp.}}{\text{hyp.}} = \frac{2}{30} \approx 0.0667$$

$$x \approx \sin^{-1} 0.0667 \approx 3.824$$

▶ The rake is about 3.8°, so it is within the suggested range of 5° or less.

✓ GUIDED PRACTICE for Examples 3 and 4

3. Solve a right triangle that has a 40° angle and a 20 inch hypotenuse.

4. **WHAT IF?** In Example 4, suppose another raked stage is 20 feet long from front to back with a total rise of 2 feet. Is this raked stage safe? *Explain*.

7.7 EXERCISES

HOMEWORK KEY
○ = **WORKED-OUT SOLUTIONS** on p. WS1 for Exs. 5, 13, and 35
★ = **STANDARDIZED TEST PRACTICE** Exs. 2, 9, 29, 30, 35, 40, and 41
◆ = **MULTIPLE REPRESENTATIONS** Ex. 39

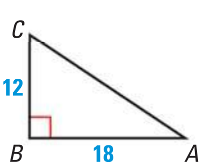
SKILL PRACTICE

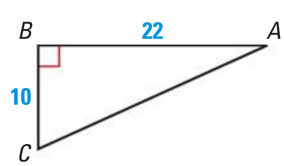
1. **VOCABULARY** Copy and complete: To solve a right triangle means to find the measures of all of its __?__ and __?__.

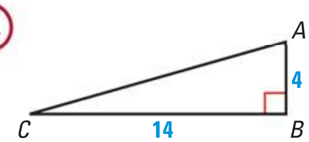
2. ★ **WRITING** *Explain* when to use a trigonometric ratio to find a side length of a right triangle and when to use the Pythagorean Theorem.

EXAMPLE 1
on p. 483 for Exs. 3–5

USING INVERSE TANGENTS **Use a calculator to approximate the measure of $\angle A$ to the nearest tenth of a degree.**

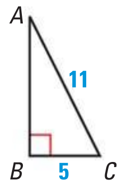
3. 

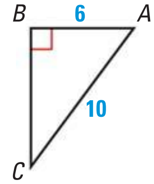
4. 

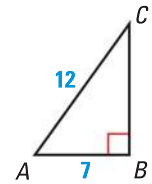
(5.) 

EXAMPLE 2
on p. 484
for Exs. 6–9

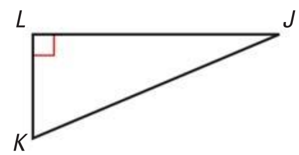
USING INVERSE SINES AND COSINES **Use a calculator to approximate the measure of $\angle A$ to the nearest tenth of a degree.**

6.


7.


8.


9. ★ **MULTIPLE CHOICE** Which expression is correct?

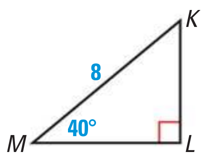


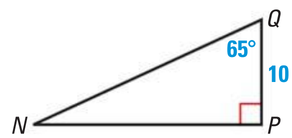
Ⓐ $\sin^{-1}\frac{JL}{JK} = m\angle J$ Ⓑ $\tan^{-1}\frac{KL}{JL} = m\angle J$

Ⓒ $\cos^{-1}\frac{JL}{JK} = m\angle K$ Ⓓ $\sin^{-1}\frac{JL}{KL} = m\angle K$

EXAMPLE 3
on p. 484
for Exs. 10–18

SOLVING RIGHT TRIANGLES **Solve the right triangle. Round decimal answers to the nearest tenth.**

10.


11.


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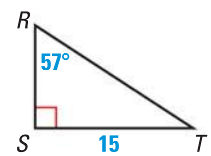

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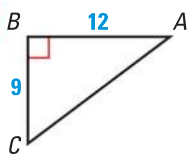

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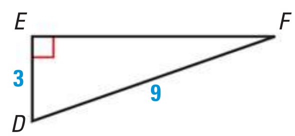

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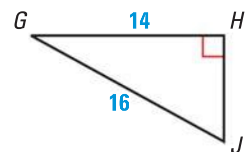

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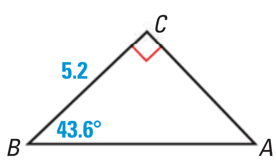
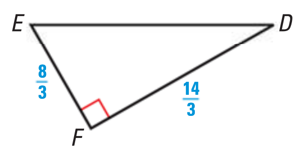
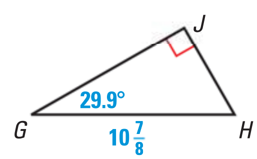
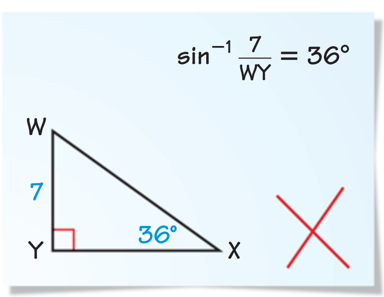
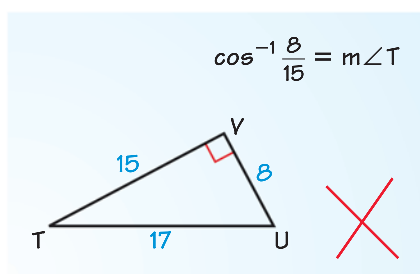


17.


18.


ERROR ANALYSIS ***Describe*** **and correct the student's error in using an inverse trigonometric ratio.**

19.


20.


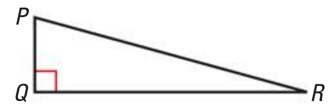
CALCULATOR **Let $\angle A$ be an acute angle in a right triangle. Approximate the measure of $\angle A$ to the nearest tenth of a degree.**

21. $\sin A = 0.5$ **22.** $\sin A = 0.75$ **23.** $\cos A = 0.33$ **24.** $\cos A = 0.64$

25. $\tan A = 1.0$ **26.** $\tan A = 0.28$ **27.** $\sin A = 0.19$ **28.** $\cos A = 0.81$

○ = WORKED-OUT SOLUTIONS on p. WS1 ★ = STANDARDIZED TEST PRACTICE

29. ★ **MULTIPLE CHOICE** Which additional information would *not* be enough to solve $\triangle PRQ$?



Ⓐ $m\angle P$ and PR Ⓑ $m\angle P$ and $m\angle R$

Ⓒ PQ and PR Ⓓ $m\angle P$ and PQ

30. ★ **WRITING** *Explain* why it is incorrect to say that $\tan^{-1} x = \frac{1}{\tan x}$.

31. **SPECIAL RIGHT TRIANGLES** If $\sin A = \frac{1}{2}\sqrt{2}$, what is $m\angle A$? If $\sin B = \frac{1}{2}\sqrt{3}$, what is $m\angle B$?

32. **TRIGONOMETRIC VALUES** Use the *Table of Trigonometric Ratios* on page 925 to answer the questions.

a. What angles have nearly the same sine and tangent values?

b. What angle has the greatest difference in its sine and tangent value?

c. What angle has a tangent value that is double its sine value?

d. Is $\sin 2x$ equal to $2 \cdot \sin x$?

33. **CHALLENGE** The perimeter of rectangle $ABCD$ is 16 centimeters, and the ratio of its width to its length is 1 : 3. Segment BD divides the rectangle into two congruent triangles. Find the side lengths and angle measures of one of these triangles.

PROBLEM SOLVING

EXAMPLE 4
on p. 485
for Exs. 34–36

34. **SOCCER** A soccer ball is placed 10 feet away from the goal, which is 8 feet high. You kick the ball and it hits the crossbar along the top of the goal. What is the angle of elevation of your kick?

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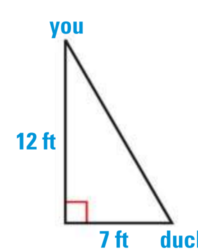
35. ★ **SHORT RESPONSE** You are standing on a footbridge in a city park that is 12 feet high above a pond. You look down and see a duck in the water 7 feet away from the footbridge. What is the angle of depression? *Explain* your reasoning.



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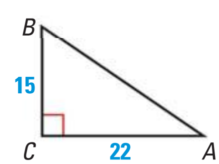
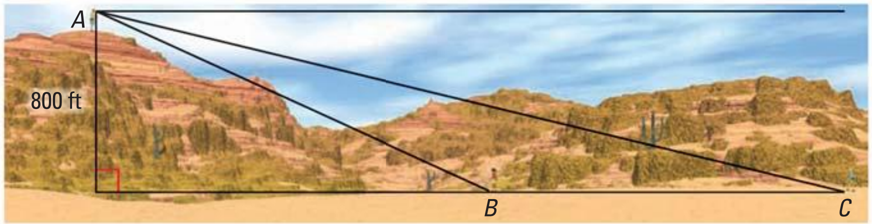
36. **CLAY** In order to unload clay easily, the body of a dump truck must be elevated to at least 55°. If the body of the dump truck is 14 feet long and has been raised 10 feet, will the clay pour out easily?

37. **REASONING** For $\triangle ABC$ shown, each of the expressions $\sin^{-1}\frac{BC}{AB}$, $\cos^{-1}\frac{AC}{AB}$, and $\tan^{-1}\frac{BC}{AC}$ can be used to approximate the measure of $\angle A$. Which expression would you choose? *Explain* your choice.



38. MULTI-STEP PROBLEM You are standing on a plateau that is 800 feet above a basin where you can see two hikers.

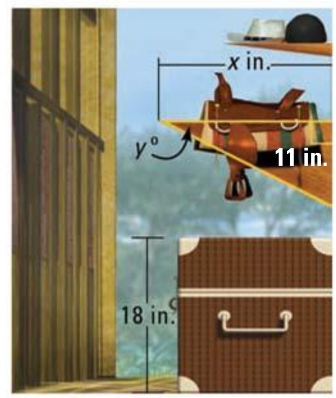


a. If the angle of depression from your line of sight to the hiker at *B* is 25°, how far is the hiker from the base of the plateau?

b. If the angle of depression from your line of sight to the hiker at *C* is 15°, how far is the hiker from the base of the plateau?

c. How far apart are the two hikers? *Explain*.

39. **MULTIPLE REPRESENTATIONS** A local ranch offers trail rides to the public. It has a variety of different sized saddles to meet the needs of horse and rider. You are going to build saddle racks that are 11 inches high. To save wood, you decide to make each rack fit each saddle.



a. Making a Table The lengths of the saddles range from 20 inches to 27 inches. Make a table showing the saddle rack length x and the measure of the adjacent angle $y°$.

b. Drawing a Graph Use your table to draw a scatterplot.

c. Making a Conjecture Make a conjecture about the relationship between the length of the rack and the angle needed.

40. ★ **OPEN-ENDED MATH** *Describe* a real-world problem you could solve using a trigonometric ratio.

41. ★ **EXTENDED RESPONSE** Your town is building a wind generator to create electricity for your school. The builder wants your geometry class to make sure that the guy wires are placed so that the tower is secure. By safety guidelines, the distance along the ground from the tower to the guy wire's connection with the ground should be between 50% to 75% of the height of the guy wire's connection with the tower.

a. The tower is 64 feet tall. The builders plan to have the distance along the ground from the tower to the guy wire's connection with the ground be 60% of the height of the tower. How far apart are the tower and the ground connection of the wire?

b. How long will a guy wire need to be that is attached 60 feet above the ground?

c. How long will a guy wire need to be that is attached 30 feet above the ground?

d. Find the angle of elevation of each wire. Are the right triangles formed by the ground, tower, and wires *congruent*, *similar*, or *neither*? *Explain*.

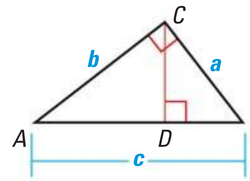
e. *Explain* which trigonometric ratios you used to solve the problem.

○ = **WORKED-OUT SOLUTIONS** on p. WS1　★ = **STANDARDIZED TEST PRACTICE**　= **MULTIPLE REPRESENTATIONS**

42. **CHALLENGE** Use the diagram of $\triangle ABC$.

GIVEN ▶ $\triangle ABC$ with altitude $\overline{CD}$.

PROVE ▶ $\frac{\sin A}{a} = \frac{\sin B}{b}$



MIXED REVIEW

PREVIEW
Prepare for Lesson 8.1 in Ex. 43.

43. Copy and complete the table. *(p. 42)*

Number of sides	Type of polygon
5	?
12	?
?	Octagon
?	Triangle
7	?

Number of sides	Type of polygon
?	*n*-gon
?	Quadrilateral
10	?
9	?
?	Hexagon

A point on an image and the transformation are given. Find the corresponding point on the original figure. *(p. 272)*

44. Point on image: (5, 1); translation: $(x, y) \to (x + 3, y - 2)$

45. Point on image: (4, −6); reflection: $(x, y) \to (x, -y)$

46. Point on image: (−2, 3); translation: $(x, y) \to (x - 5, y + 7)$

Draw a dilation of the polygon with the given vertices using the given scale factor *k*. *(p. 409)*

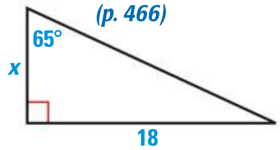
47. $A(2, 2), B(-1, -3), C(5, -3); k = 2$

48. $A(-4, -2), B(-2, 4), C(3, 6), D(6, 3); k = \frac{1}{2}$

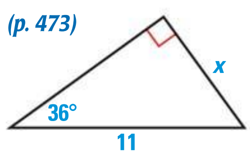
QUIZ *for Lessons 7.5–7.7*

Find the value of *x* to the nearest tenth.

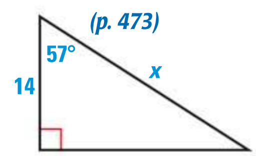
1. *(p. 466)*



2. *(p. 473)*

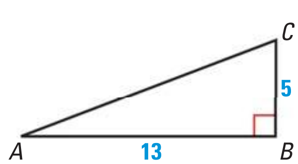


3. *(p. 473)*

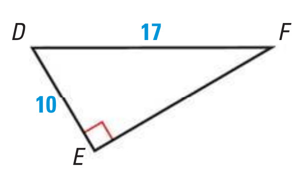


Solve the right triangle. Round decimal answers to the nearest tenth. *(p. 483)*

4.



5.



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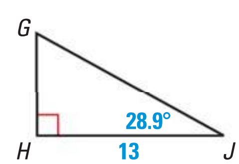


Extension

Use after Lesson 7.7

Law of Sines and Law of Cosines

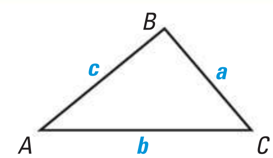
GOAL Use trigonometry with acute and obtuse triangles.

The trigonometric ratios you have seen so far in this chapter can be used to find angle and side measures in right triangles. You can use the Law of Sines to find angle and side measures in *any* triangle.

KEY CONCEPT — *For Your Notebook*

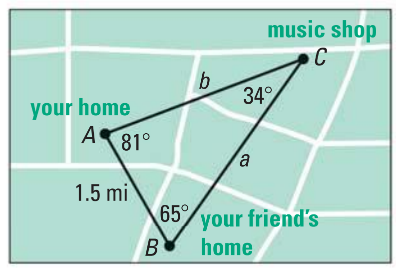
Law of Sines

If $\triangle ABC$ has sides of length a, b, and c as shown, then $\frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c}$.



EXAMPLE 1 Find a distance using Law of Sines

DISTANCE Use the information in the diagram to determine how much closer you live to the music store than your friend does.



Solution

STEP 1 **Use** the Law of Sines to find the distance a from your friend's home to the music store.

$\frac{\sin A}{a} = \frac{\sin C}{c}$ **Write Law of Sines.**

$\frac{\sin 81°}{a} = \frac{\sin 34°}{1.5}$ **Substitute.**

$a \approx 2.6$ **Solve for *a*.**

STEP 2 **Use** the Law of Sines to find the distance b from your home to the music store.

$\frac{\sin B}{b} = \frac{\sin C}{c}$ **Write Law of Sines.**

$\frac{\sin 65°}{b} = \frac{\sin 34°}{1.5}$ **Substitute.**

$b \approx 2.4$ **Solve for *b*.**

STEP 3 **Subtract** the distances.

$a - b \approx 2.6 - 2.4 = 0.2$

▶ You live about 0.2 miles closer to the music store.

LAW OF COSINES You can also use the Law of Cosines to solve any triangle.

KEY CONCEPT — *For Your Notebook*

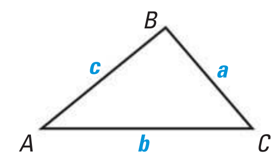
Law of Cosines

If $\triangle ABC$ has sides of length a, b, and c, then:

$$a^2 = b^2 + c^2 - 2bc \cos A$$

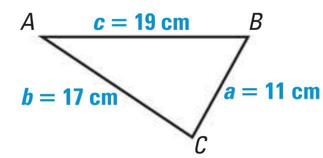
$$b^2 = a^2 + c^2 - 2ac \cos B$$

$$c^2 = a^2 + b^2 - 2ab \cos C$$



EXAMPLE 2 Find an angle measure using Law of Cosines

In $\triangle ABC$ at the right, $a = 11$ cm, $b = 17$ cm, and $c = 19$ cm. Find $m\angle C$.



Solution

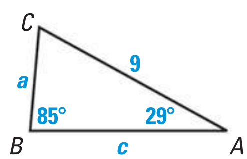
$c^2 = a^2 + b^2 - 2ab \cos C$	**Write Law of Cosines.**
$19^2 = 11^2 + 17^2 - 2(11)(17) \cos C$	**Substitute.**
$0.1310 = \cos C$	**Solve for cos C.**
$m\angle C \approx 82°$	**Find $\cos^{-1}$ (0.1310).**

PRACTICE

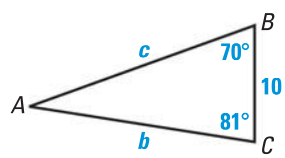
EXAMPLE 1 for Exs. 1–3

LAW OF SINES Use the Law of Sines to solve the triangle. Round decimal answers to the nearest tenth.

1.



2.



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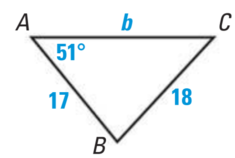
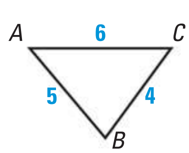


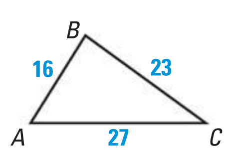
EXAMPLE 2 for Exs. 4–7

LAW OF COSINES Use the Law of Cosines to solve the triangle. Round decimal answers to the nearest tenth.

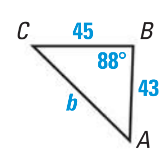
4.



5.



6.



7. DISTANCE Use the diagram at the right. Find the straight distance between the zoo and movie theater.